

# Homework 1

2022 年 9 月 5、9 日布置

2022 年 9 月 16 日交

## 1 计算度规

Gauss 假定，在空间任一足够小的区域内能找到一个局部的欧氏坐标系  $(\xi_1, \xi_2)$ ，使得两点  $(\xi_1, \xi_2)$  和  $(\xi_1 + d\xi_1, \xi_2 + d\xi_2)$  间的距离为  $ds^2 = d\xi_1^2 + d\xi_2^2$ 。对于非欧几何，该坐标不能覆盖有限部分，选择另外能覆盖全空间的坐标系  $(x_1, x_2)$ ，计算出

$$ds^2 = d\xi_1^2 + d\xi_2^2 = g_{11}dx_1^2 + 2g_{12}dx_1dx_2 + g_{22}dx_2^2$$

推导出其中  $g_{ij}$  的表达式。(结果为  $g_{11} = \left(\frac{\partial \xi_1}{\partial x_1}\right)^2 + \left(\frac{\partial \xi_2}{\partial x_1}\right)^2$ ,  $g_{12} = \frac{\partial \xi_1}{\partial x_1} \frac{\partial \xi_1}{\partial x_2} + \frac{\partial \xi_2}{\partial x_1} \frac{\partial \xi_2}{\partial x_2}$ ,  $g_{22} = \left(\frac{\partial \xi_1}{\partial x_2}\right)^2 + \left(\frac{\partial \xi_2}{\partial x_2}\right)^2$ )

## 2 Gauss 曲率

$$\begin{aligned} K = & \frac{1}{2g} \left[ 2 \frac{\partial^2 g_{12}}{\partial x_1 \partial x_2} - \frac{\partial^2 g_{11}}{\partial x_2^2} - \frac{\partial^2 g_{22}}{\partial x_1^2} \right] \\ & - \frac{g_{22}}{4g^2} \left[ \left( \frac{\partial g_{11}}{\partial x_1} \right) \left( 2 \frac{\partial g_{12}}{\partial x_2} - \frac{\partial g_{22}}{\partial x_1} \right) - \left( \frac{\partial g_{11}}{\partial x_2} \right)^2 \right] \\ & + \frac{g_{12}}{4g^2} \left[ \left( \frac{\partial g_{11}}{\partial x_1} \right) \left( \frac{\partial g_{22}}{\partial x_2} \right) - 2 \left( \frac{\partial g_{11}}{\partial x_2} \right) \left( \frac{\partial g_{22}}{\partial x_1} \right) + \left( 2 \frac{\partial g_{12}}{\partial x_1} - \frac{\partial g_{11}}{\partial x_2} \right) \left( 2 \frac{\partial g_{12}}{\partial x_2} - \frac{\partial g_{22}}{\partial x_1} \right) \right] \\ & - \frac{g_{11}}{4g^2} \left[ \left( \frac{\partial g_{22}}{\partial x_2} \right) \left( 2 \frac{\partial g_{12}}{\partial x_1} - \frac{\partial g_{11}}{\partial x_2} \right) - \left( \frac{\partial g_{22}}{\partial x_1} \right)^2 \right] \end{aligned}$$

其中  $g$  为度规的行列式  $g = g_{11}g_{22} - g_{12}^2$

对于球面  $ds^2 = a^2(d\theta^2 + \sin^2\theta d\phi^2)$ ，计算  $K$  值，写出每一项计算过程。答案应为  $K = \frac{1}{a^2}$

## 3 引力半径

地球质量为  $m = 5.977 \times 10^{24} \text{kg}$ ，计算地球的引力半径